

# EDI 2010/2

## Prova 4

### Soluções de EDOs em Séries de Potências em torno de pontos ordinários

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1. Determine a solução geral da equação diferencial dada que é válida em qualquer intervalo que não inclui a singularidade.

(i)  $x^2 y'' + 4xy' + 2y = 0$  ,                      (ii)  $x^2 y'' - 3xy' + 4y = 0$  ,

Solução

Ambas equações são de Euler-Cauchy.

(i)

```
> restart
> eq1 := x^2·diff(y(x), x$2) + 4·x·diff(y(x), x) + 2·y(x) = 0
      eq1 := x^2 ⎛⎜  $\frac{d^2}{dx^2} y(x)$  ⎞⎟ + 4x ⎛⎜  $\frac{d}{dx} y(x)$  ⎞⎟ + 2y(x) = 0
```

 (1.1)

```
> dsolve(eq1, y(x))
      y(x) =  $\frac{C1}{x^2} + \frac{C2}{x}$ 
```

 (1.2)

(ii)

```
> eq2 := x^2·diff(y(x), x$2) - 3·x·diff(y(x), x) + 4·y(x) = 0
      eq2 := x^2 ⎛⎜  $\frac{d^2}{dx^2} y(x)$  ⎞⎟ - 3x ⎛⎜  $\frac{d}{dx} y(x)$  ⎞⎟ + 4y(x) = 0
```

 (1.3)

```
> dsolve(eq2, y(x))
      y(x) =  $\_C1 x^2 + \_C2 x^2 \ln(x)$ 
```

 (1.4)

2. Nos problema abaixo mostre que a dada equação diferencial tem um ponto singular regular em  $x = 0$  e determine duas soluções linearmente independentes para  $x > 0$ .

(i)  $x^2 y'' + 2xy' + xy = 0$  ,                      (ii)  $x^2 y'' + xy' + 2xy = 0$

Solução

```
(i)
> restart;
> P := x^2: Q:=2*x: R:=x:
> S2 := sum(simplify(P*c[n]*(n+r)*(n+r-1)*x^(n+r-2)), n = 0 ..
```

**k);**

$$S2 := \sum_{n=0}^k x^{n+r} c_n (n+r) (n+r-1) \quad (2.1.1)$$

**> S1 := sum(simplify(Q\*c[n]\*(n+r)\*x^(n+r-1)), n = 0 .. k);**

$$S1 := \sum_{n=0}^k 2 x^{n+r} c_n (n+r) \quad (2.1.2)$$

**> S0 := sum(simplify(R\*c[n]\*x^(n+r)), n = 0 .. k);**

$$S0 := \sum_{n=0}^k x^{n+1+r} c_n \quad (2.1.3)$$

A equação pode agora ser escrita como

**> S := S2+S1+S0;**

$$S := \sum_{n=0}^k x^{n+r} c_n (n+r) (n+r-1) + \sum_{n=0}^k 2 x^{n+r} c_n (n+r) + \sum_{n=0}^k x^{n+1+r} c_n \quad (2.1.4)$$

**> k := 6;**

$$k := 6 \quad (2.1.5)$$

**> eq1 := simplify(S/x^r);**

$$\begin{aligned} eq1 := & x^5 c_4 + 3 x c_1 r + x c_1 r^2 + 5 x^2 c_2 r + x^2 c_2 r^2 + 7 x^3 c_3 r + x^3 c_3 r^2 + 9 x^4 c_4 r \\ & + x^4 c_4 r^2 + 11 x^5 c_5 r + x^5 c_5 r^2 + 13 x^6 c_6 r + x^6 c_6 r^2 + x^6 c_5 + x c_0 + c_0 r^2 \\ & + c_0 r + x^7 c_6 + 6 x^2 c_2 + 12 x^3 c_3 + 20 x^4 c_4 + 30 x^5 c_5 + 42 x^6 c_6 + 2 x c_1 \\ & + x^2 c_1 + x^3 c_2 + x^4 c_3 \end{aligned} \quad (2.1.6)$$

**> eq2 := x\*%:**

**> eq3 := collect(%, x);**

$$\begin{aligned} eq3 := & c_6 x^8 + (c_6 r^2 + 42 c_6 + 13 c_6 r + c_5) x^7 + (11 c_5 r + c_4 + 30 c_5 + c_5 r^2) x^6 \\ & + (c_3 + c_4 r^2 + 9 c_4 r + 20 c_4) x^5 + (c_2 + c_3 r^2 + 12 c_3 + 7 c_3 r) x^4 + (c_2 r^2 \\ & + 5 c_2 r + 6 c_2 + c_1) x^3 + (3 c_1 r + 2 c_1 + c_1 r^2 + c_0) x^2 + (c_0 r^2 + c_0 r) x \end{aligned} \quad (2.1.7)$$

**> e0 := coeff(eq3, x, 1);**

$$e0 := c_0 r^2 + c_0 r \quad (2.1.8)$$

**> s := solve(e0, r);**

$$s := 0, -1 \quad (2.1.9)$$

**> r1 := s[1]; r2 := s[2];**

$$\begin{aligned} r1 & := 0 \\ r2 & := -1 \end{aligned} \quad (2.1.10)$$

Como  $r1 - r2 = 1$ , as relações de recorrências associadas a cada raiz não geram soluções linearmente independentes. Geremos a primeira solução

**> r := r1;**

**> eqs := array(2 .. 6);**

```

for n from 2 to 6 do
  eqs[n] := coeff(eq3, x, n) = 0
end do;
coeffEqs := convert(eqs, list);
eqs := array(2..6, [ ])
eqs_2 := 2 c_1 + c_0 = 0
eqs_3 := 6 c_2 + c_1 = 0
eqs_4 := c_2 + 12 c_3 = 0
eqs_5 := c_3 + 20 c_4 = 0
eqs_6 := c_4 + 30 c_5 = 0

```

$$\text{coeffEqs} := [2 c_1 + c_0 = 0, 6 c_2 + c_1 = 0, c_2 + 12 c_3 = 0, c_3 + 20 c_4 = 0, c_4 + 30 c_5 = 0] \quad (2.1.11)$$

Temos aqui 6 equações para 6 incógnitas  $c_1$  a  $c_5$ .

```

> s1 := [seq(c[n], n = 1 .. 5)];
s1 := [c_1, c_2, c_3, c_4, c_5]

```

Resolvendo estas equações para  $a_0$  obtemos

```

> coef := solve(coeffEqs, s1);
coef := [[c_1 = -1/2 c_0, c_2 = 1/12 c_0, c_3 = -1/144 c_0, c_4 = 1/2880 c_0, c_5 = -1/86400 c_0]]

```

```

> assign(coef)
> y1 := expand(x^r*(sum(c[i]*x^i, i = 0 .. 5))/c[0]);
y1 := 1 - 1/2 x + 1/12 x^2 - 1/144 x^3 + 1/2880 x^4 - 1/86400 x^5

```

Encontremos agora a segunda solução. Como  $r1 - r2 = 1 > 0$ , devemos buscar uma solução da forma:

$$y2 = B y1 \ln x + x^{r2} \sum_{k=0}^{\infty} b_k x^k$$

```

> r := r2
r := -1

```

```

> y2 := B*y1*ln(x)+x^r2*(sum(b[j]*x^j, j = 0 .. 6));
y2 := B (1 - 1/2 x + 1/12 x^2 - 1/144 x^3 + 1/2880 x^4 - 1/86400 x^5) ln(x)
+ (b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6) / x

```

Se  $B = 0$ ,

```

> coeff(eq3, x, 2)
c[0]

```

O que implicaria  $c_0 = 0$ , de modo que devemos supor que  $B \neq 0$ .

```
> eq4:=x^2*(diff(y2, x, x))+2*x*(diff(y2, x))+x*y2:
> eq5 := expand(eq4)
```

$$eq5 := B - \frac{1}{86400} x^6 B \ln(x) - \frac{11}{86400} x^5 B + \frac{1}{320} x^4 B - \frac{7}{144} B x^3 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6 + 2 b_2 x + 6 b_3 x^2 + 12 b_4 x^3 + 20 b_5 x^4 + 30 b_6 x^5 - \frac{3}{2} B x + \frac{5}{12} B x^2 + b_0 \quad (2.1.18)$$

O termo envolvendo logaritmo deve ser desprezado, pois envolve uma potência além de 5. Devemos agora igualar a zero os coeficientes de  $x$ :

```
> eq6 := expand((eq5 + \frac{1}{86400} B \ln(x) x^6))
```

$$eq6 := B - \frac{11}{86400} x^5 B + \frac{1}{320} x^4 B - \frac{7}{144} B x^3 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6 + 2 b_2 x + 6 b_3 x^2 + 12 b_4 x^3 + 20 b_5 x^4 + 30 b_6 x^5 - \frac{3}{2} B x + \frac{5}{12} B x^2 + b_0 \quad (2.1.19)$$

```
> eqs2 := array(1 .. 6);
for n from 1 to 6 do
    eqs2[n] := coeff(eq6, x, n-1) = 0
end do;
coeffEqs := convert(eqs2, list);
eqs2 := array(1..6, [ ])
eqs2_1 := B + b_0 = 0
eqs2_2 := b_1 + 2 b_2 - \frac{3}{2} B = 0
eqs2_3 := b_2 + 6 b_3 + \frac{5}{12} B = 0
eqs2_4 := -\frac{7}{144} B + b_3 + 12 b_4 = 0
eqs2_5 := \frac{1}{320} B + b_4 + 20 b_5 = 0
eqs2_6 := -\frac{11}{86400} B + b_5 + 30 b_6 = 0
```

$$coeffEqs := \left[ B + b_0 = 0, b_1 + 2 b_2 - \frac{3}{2} B = 0, b_2 + 6 b_3 + \frac{5}{12} B = 0, -\frac{7}{144} B + b_3 + 12 b_4 = 0, \frac{1}{320} B + b_4 + 20 b_5 = 0, -\frac{11}{86400} B + b_5 + 30 b_6 = 0 \right] \quad (2.1.20)$$

```
> s2 := [B, seq(b[n], n = 1 .. 6)];
s2 := [B, b_1, b_2, b_3, b_4, b_5, b_6] \quad (2.1.21)
```

> sol2 := solve(coeffEqs, s2)

$$\text{sol2} := \left[ \left[ B = -b_0, b_1 = -\frac{56}{15} b_0 - 86400 b_6, b_2 = \frac{67}{60} b_0 + 43200 b_6, b_3 = -\frac{7}{60} b_0 \right. \right. \quad (2.1.22)$$

$$\left. \left. - 7200 b_6, b_4 = \frac{49}{8640} b_0 + 600 b_6, b_5 = -\frac{11}{86400} b_0 - 30 b_6, b_6 = b_6 \right] \right]$$

> assign(sol2)

Como devemos obter somente uma solução particular.

>  $b_0 := 1; b_6 := \frac{\frac{56}{15}}{-86400}$

$$b_0 := 1$$

$$b_6 := -\frac{7}{162000} \quad (2.1.23)$$

>  $b_1$

$$0$$

(2.1.24)

> yy2 := expand(y2)

$$\text{yy2} := -\ln(x) + \frac{1}{2} \ln(x) x - \frac{1}{12} \ln(x) x^2 + \frac{1}{144} \ln(x) x^3 - \frac{1}{2880} \ln(x) x^4 \quad (2.1.25)$$

$$+ \frac{1}{86400} \ln(x) x^5 + \frac{1}{x} - \frac{3}{4} x + \frac{7}{36} x^2 - \frac{35}{1728} x^3 + \frac{101}{86400} x^4$$

$$- \frac{7}{162000} x^5$$

A solução geral é então,

> **Y := C[1]\*y1+C[2]\*yy2;**

$$Y := C_1 \left( 1 - \frac{1}{2} x + \frac{1}{12} x^2 - \frac{1}{144} x^3 + \frac{1}{2880} x^4 - \frac{1}{86400} x^5 \right) + C_2 \left( -\ln(x) \right) \quad (2.1.26)$$

$$+ \frac{1}{2} \ln(x) x - \frac{1}{12} \ln(x) x^2 + \frac{1}{144} \ln(x) x^3 - \frac{1}{2880} \ln(x) x^4$$

$$+ \frac{1}{86400} \ln(x) x^5 + \frac{1}{x} - \frac{3}{4} x + \frac{7}{36} x^2 - \frac{35}{1728} x^3 + \frac{101}{86400} x^4$$

$$- \frac{7}{162000} x^5$$

Resolvamos esta equação diretamente, com o comando dsolve:

> Order := 7:

> eq := x^2 \* (diff(Y1(x), x, x)) + 2 \* x \* (diff(Y1(x), x)) + x \* Y1(x) = 0;

$$\text{eq} := x^2 \left( \frac{d^2}{dx^2} Y1(x) \right) + 2x \left( \frac{d}{dx} Y1(x) \right) + x Y1(x) = 0 \quad (2.1.27)$$

> dsolve(eq, Y1(x), series)

$$Y1(x) = \_C1 \left( 1 - \frac{1}{2} x + \frac{1}{12} x^2 - \frac{1}{144} x^3 + \frac{1}{2880} x^4 - \frac{1}{86400} x^5 \right) \quad (2.1.28)$$

$$\begin{aligned}
& + \frac{1}{3628800} x^6 + O(x^7) \Big) \\
& + \_C2 \left( \frac{1}{x} \left( \ln(x) \left( -x + \frac{1}{2} x^2 - \frac{1}{12} x^3 + \frac{1}{144} x^4 - \frac{1}{2880} x^5 \right. \right. \right. \\
& \left. \left. \left. + \frac{1}{86400} x^6 + O(x^7) \right) \right) \right) \\
& + \frac{1 - \frac{3}{4} x^2 + \frac{7}{36} x^3 - \frac{35}{1728} x^4 + \frac{101}{86400} x^5 - \frac{7}{162000} x^6 + O(x^7)}{x} \Big)
\end{aligned}$$

(ii)

```

> restart;
> P := x^2: Q:=x: R:=2*x:
> S2 := sum(simplify(P*c[n]*(n+r)*(n+r-1)*x^(n+r-2)), n = 0 ..
k);

```

$$S2 := \sum_{n=0}^k x^{n+r} c_n (n+r) (n+r-1) \quad (2.2.1)$$

```

> S1 := sum(simplify(Q*c[n]*(n+r)*x^(n+r-1)), n = 0 .. k);

```

$$S1 := \sum_{n=0}^k x^{n+r} c_n (n+r) \quad (2.2.2)$$

```

> S0 := sum(simplify(R*c[n]*x^(n+r)), n = 0 .. k);

```

$$S0 := \sum_{n=0}^k 2 x^{n+1+r} c_n \quad (2.2.3)$$

A equação pode agora ser escrita como

```

> S := S2+S1+S0;

```

$$S := \sum_{n=0}^k x^{n+r} c_n (n+r) (n+r-1) + \sum_{n=0}^k x^{n+r} c_n (n+r) + \sum_{n=0}^k 2 x^{n+1+r} c_n \quad (2.2.4)$$

```

> k := 6;

```

$$k := 6 \quad (2.2.5)$$

```

> eq1 := simplify(S/x^r);

```

$$\begin{aligned}
eq1 := & x c_1 r^2 + 4 x^2 c_2 r + x^2 c_2 r^2 + 6 x^3 c_3 r + x^3 c_3 r^2 + 8 x^4 c_4 r + x^4 c_4 r^2 \\
& + 10 x^5 c_5 r + x^5 c_5 r^2 + 12 x^6 c_6 r + x^6 c_6 r^2 + 2 x^7 c_6 + c_0 r^2 + 4 x^2 c_2 \\
& + 9 x^3 c_3 + 16 x^4 c_4 + 25 x^5 c_5 + 36 x^6 c_6 + x c_1 + 2 x c_0 + 2 x^2 c_1 + 2 x^3 c_2 \\
& + 2 x^4 c_3 + 2 x^5 c_4 + 2 x^6 c_5 + 2 x c_1 r
\end{aligned} \quad (2.2.6)$$

```

> eq2 := x*%:

```

```
> eq3 := collect(%, x);
```

$$eq3 := 2 c_6 x^8 + (2 c_5 + 12 c_6 r + c_6 r^2 + 36 c_6) x^7 + (25 c_5 + 2 c_4 + 10 c_5 r + c_5 r^2) x^6 + (16 c_4 + 8 c_4 r + c_4 r^2 + 2 c_3) x^5 + (6 c_3 r + c_3 r^2 + 9 c_3 + 2 c_2) x^4 + (2 c_1 + c_2 r^2 + 4 c_2 r + 4 c_2) x^3 + (c_1 + 2 c_0 + c_1 r^2 + 2 c_1 r) x^2 + c_0 r^2 x \quad (2.2.7)$$

```
> e0 := coeff(eq3, x, 1);
```

$$e0 := c_0 r^2 \quad (2.2.8)$$

```
> s := solve(e0, r);
```

$$s := 0, 0 \quad (2.2.9)$$

```
> r1 := s[1]; r2 := s[2];
```

$$\begin{aligned} r1 &:= 0 \\ r2 &:= 0 \end{aligned} \quad (2.2.10)$$

Como  $r1 - r2 = 1$ , as relações de recorrências associadas a cada raiz não geram soluções linearmente independentes. Geremos a primeira solução

```
> r := r1;
```

```
> eqs := array(2 .. 6);
```

```
for n from 2 to 6 do
```

```
    eqs[n] := coeff(eq3, x, n) = 0
```

```
end do;
```

```
coeffEqs := convert(eqs, list);
```

$$\begin{aligned} eqs_2 &:= c_1 + 2 c_0 = 0 \\ eqs_3 &:= 2 c_1 + 4 c_2 = 0 \\ eqs_4 &:= 9 c_3 + 2 c_2 = 0 \\ eqs_5 &:= 16 c_4 + 2 c_3 = 0 \\ eqs_6 &:= 25 c_5 + 2 c_4 = 0 \end{aligned}$$

$$coeffEqs := [c_1 + 2 c_0 = 0, 2 c_1 + 4 c_2 = 0, 9 c_3 + 2 c_2 = 0, 16 c_4 + 2 c_3 = 0, 25 c_5 + 2 c_4 = 0] \quad (2.2.11)$$

Temos aqui 6 equações para 6 incógnitas  $c_1$  a  $c_5$ .

```
> s1 := [seq(c[n], n = 1 .. 5)];
```

$$s1 := [c_1, c_2, c_3, c_4, c_5] \quad (2.2.12)$$

```
>
```

Resolvendo estas equações para  $a_0$  obtemos

```
> coef := solve(coeffEqs, s1);
```

$$coef := \left[ \left[ c_1 = -2 c_0, c_2 = c_0, c_3 = -\frac{2}{9} c_0, c_4 = \frac{1}{36} c_0, c_5 = -\frac{1}{450} c_0 \right] \right] \quad (2.2.13)$$

```

> assign(coef)
> y1 := expand(x^r*(sum(c[i]*x^i, i = 0 .. 5))/c[0]);

```

$$y1 := 1 - 2x + x^2 - \frac{2}{9}x^3 + \frac{1}{36}x^4 - \frac{1}{450}x^5 \quad (2.2.14)$$

Encontremos agora a segunda solução. Como  $r1 - r2 = 0$ , devemos buscar uma solução da forma:

$$y2 = y1 \ln x + x^{r2} \sum_{k=0}^{\infty} b_k x^k$$

```

> r := r2

```

$$r := 0 \quad (2.2.15)$$

```

> y2 := y1*ln(x)+x^r2*(sum(b[j]*x^j, j = 0 .. 5));

```

$$y2 := \left(1 - 2x + x^2 - \frac{2}{9}x^3 + \frac{1}{36}x^4 - \frac{1}{450}x^5\right) \ln(x) + b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 + b_5x^5 \quad (2.2.16)$$

Substituindo na equação diferencial obtemos

```

> eq4:=x^2*(diff(y2, x, x))+x*(diff(y2, x))+2*x*y2:
> eq5 := expand(eq4)

```

$$eq5 := -4x + 4x^2 - \frac{4}{3}x^3 + \frac{2}{9}x^4 - \frac{1}{45}x^5 + b_1x + 4b_2x^2 + 9b_3x^3 + 16b_4x^4 + 25b_5x^5 - \frac{1}{225}x^6 \ln(x) + 2xb_0 + 2b_1x^2 + 2b_2x^3 + 2b_3x^4 + 2b_4x^5 + 2b_5x^6 \quad (2.2.17)$$

O termo envolvendo logaritmo deve ser desprezado, pois envolve uma potência além de 5.

Devemos agora igualar a zero os coeficientes de  $x$ :

```

> eq6 := expand((eq5 + \frac{1}{225}x^6 \ln(x)))

```

$$eq6 := -4x + 4x^2 - \frac{4}{3}x^3 + \frac{2}{9}x^4 - \frac{1}{45}x^5 + b_1x + 4b_2x^2 + 9b_3x^3 + 16b_4x^4 + 25b_5x^5 + 2xb_0 + 2b_1x^2 + 2b_2x^3 + 2b_3x^4 + 2b_4x^5 + 2b_5x^6 \quad (2.2.18)$$

```

> eqs2 := array(1 .. 5);
for n from 1 to 5 do
    eqs2[n] := coeff(eq6, x, n) = 0
end do;
coeffEqs := convert(eqs2, list);

```

$$eqs2 := array(1..5, [ ])$$

$$eqs2_1 := -4 + b_1 + 2b_0 = 0$$

$$eqs2_2 := 4 + 4b_2 + 2b_1 = 0$$

$$eqs2_3 := -\frac{4}{3} + 9b_3 + 2b_2 = 0$$

$$eqs2_4 := \frac{2}{9} + 16b_4 + 2b_3 = 0$$

$$eqs2_5 := -\frac{1}{45} + 25 b_5 + 2 b_4 = 0$$

$$coeffEqs := \left[ -4 + b_1 + 2 b_0 = 0, 4 + 4 b_2 + 2 b_1 = 0, -\frac{4}{3} + 9 b_3 + 2 b_2 = 0, \frac{2}{9} + 16 b_4 + 2 b_3 = 0, -\frac{1}{45} + 25 b_5 + 2 b_4 = 0 \right] \quad (2.2.19)$$

$$\begin{aligned} > s2 := [seq(b[n], n = 1 .. 5)]; \\ & \quad \quad \quad s2 := [b_1, b_2, b_3, b_4, b_5] \end{aligned} \quad (2.2.20)$$

$$> sol2 := solve(coeffEqs, s2)$$

$$\begin{aligned} sol2 := & \left[ \left[ b_1 = 4 - 2 b_0, b_2 = -3 + b_0, b_3 = \frac{22}{27} - \frac{2}{9} b_0, b_4 = -\frac{25}{216} + \frac{1}{36} b_0, b_5 \right. \right. \\ & \left. \left. = \frac{137}{13500} - \frac{1}{450} b_0 \right] \right] \end{aligned} \quad (2.2.21)$$

$$> assign(sol2)$$

Como devemos obter somente uma solução particular,

$$> b_0 := 0;$$

$$b_0 := 0 \quad (2.2.22)$$

$$> yy2 := expand(y2)$$

$$\begin{aligned} yy2 := & \ln(x) - 2 \ln(x) x + \ln(x) x^2 - \frac{2}{9} \ln(x) x^3 + \frac{1}{36} x^4 \ln(x) - \frac{1}{450} x^5 \ln(x) \\ & + 4x - 3x^2 + \frac{22}{27} x^3 - \frac{25}{216} x^4 + \frac{137}{13500} x^5 \end{aligned} \quad (2.2.23)$$

A solução geral é então,

$$> \mathbf{Y := C[1]*y1+C[2]*yy2;}$$

$$\begin{aligned} Y := & C_1 \left( 1 - 2x + x^2 - \frac{2}{9} x^3 + \frac{1}{36} x^4 - \frac{1}{450} x^5 \right) + C_2 \left( \ln(x) - 2 \ln(x) x \right. \\ & + \ln(x) x^2 - \frac{2}{9} \ln(x) x^3 + \frac{1}{36} x^4 \ln(x) - \frac{1}{450} x^5 \ln(x) + 4x - 3x^2 \\ & \left. + \frac{22}{27} x^3 - \frac{25}{216} x^4 + \frac{137}{13500} x^5 \right) \end{aligned} \quad (2.2.24)$$

Resolvamos esta equação diretamente, com o comando dsolve:

$$> Order := 6:$$

$$> eq := x^2 * (diff(YI(x), x, x)) + x * (diff(YI(x), x)) + 2 * x * YI(x) = 0;$$

$$eq := x^2 \left( \frac{d^2}{dx^2} YI(x) \right) + x \left( \frac{d}{dx} YI(x) \right) + 2 x YI(x) = 0 \quad (2.2.25)$$

$$> dsolve(eq, YI(x), series)$$

$$YI(x) = \_C1 \left( 1 - 2x + x^2 - \frac{2}{9} x^3 + \frac{1}{36} x^4 - \frac{1}{450} x^5 + O(x^6) \right) \quad (2.2.26)$$

$$\left[ \left[ \begin{aligned} &+ C_2 \left( \ln(x) \left( 1 - 2x + x^2 - \frac{2}{9}x^3 + \frac{1}{36}x^4 - \frac{1}{450}x^5 + O(x^6) \right) + \left( 4x \right. \right. \right. \\ &\left. \left. \left. - 3x^2 + \frac{22}{27}x^3 - \frac{25}{216}x^4 + \frac{137}{13500}x^5 + O(x^6) \right) \right) \right] \end{aligned} \right]$$